Tutorial 9

Resilient Communication Systems
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MIMO Capacity I

Problem 1

Information theory background:

- Prove that Gaussian distribution leads to the maximum entropy distribution with a constant variance σ^2.
- Prove that the only function satisfying the following four properties is $I(X_i) = \log(\frac{1}{\Pr(X_i)})$.
  - $I(X_i) \geq 0$.
  - $I(X_i)$ is monotonically decreasing in $\Pr(X_i)$.
  - $I(X_i) = 0$ if $\Pr(X_i) = 1$.
  - $I(X_i, X_j) = I(X_i) + I(X_j)$ if and only if X_i and X_j are independent

Problem 2

SISO channel capacity:
Consider a single-input single-output (SISO) communication system operating over a frequency-selective time-invariant channel. The overall operation bandwidth is $B = 30$ MHz and can be divided into 3 sub-channels in which the channel coefficients are constant over frequency (e.g., by using OFDM). Let these channel coefficients be: $h_1 = 1$, $h_2 = 0.5$, and $h_3 = 0.3$. Assuming a transmit power of $P_{\mathsf{TX}} = 1$ mW and additive white Gaussian noise (AWGN) with power spectral density of $N_0 = 10^{-11}$ W/Hz, calculate the Shannon capacity.
Hint: Use the water-filling algorithm for optimum power allocation.

Problem 3

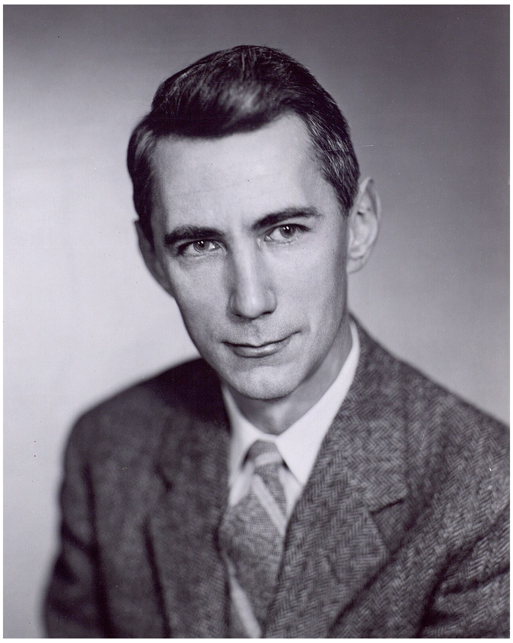
MIMO channel capacity with CSIT and CSIR:
Consider a MIMO system with the following channel matrix:

$$\mathbf{H} = \begin{bmatrix} 0.2 & -0.2 & 0.2 \\ 0.1 & 0.4 & -1 \\ 1 & 0.2 & 1 \end{bmatrix}. \qquad (1)$$

Suppose that SNR=10 dB, $B = 1$ MHz, and that the channel is perfectly known to the transmitter (CSIT) and the receiver (CSIR). Calculate the Shannon capacity using the water-filling power allocation algorithm.

Claude Elwood Shannon

Claude Elwood Shannon was a pioneering American mathematician, electrical engineer, and cryptographer, widely regarded as the "Father of Information Theory." Born on April 30, 1916, in Petoskey, Michigan, Shannon demonstrated an early aptitude for science and mathematics. He earned a B.S. in Electrical Engineering and Mathematics from the University of Michigan in 1936 and an M.S. in Electrical Engineering from the Massachusetts Institute of Technology (MIT) in 1937. He later completed his Ph.D. in Mathematics at MIT in 1940.



Claude Elwood Shannon
Born: April 30, 1916
Died: February 24, 2001 (aged 84)

Shannon began his groundbreaking career as a research assistant at MIT and later joined Bell Telephone Laboratories, where he developed many of his seminal ideas. In 1948, Shannon published his landmark paper, A Mathematical Theory of Communication, which introduced the concepts of information entropy, channel capacity, and the quantification of information [1]. This work laid the foundation for modern digital communication and data compression, enabling innovations such as the internet, cellular networks, and digital storage.

One of Shannon's key contributions was the establishment of the binary system as the optimal method for representing and transmitting information. By connecting Boolean algebra with electrical circuits, he revolutionized the design of digital computers and communication systems [2]. His work bridged the fields of mathematics, engineering, and computer science, profoundly influencing each discipline [3].

Throughout his illustrious career, Shannon received numerous awards and honors, including the National Medal of Science in 1966 and the IEEE Medal of Honor in 1966. He was also elected to the National Academy of Sciences and the National Academy of Engineering.

In addition to his scientific brilliance, Shannon was known for his playful curiosity and inventive spirit. He built machines like a mechanical mouse capable of navigating a maze and a juggling robot, showcasing his creative approach to problem-solving.

Claude Shannon's work continues to underpin the digital age, shaping the way information is stored, transmitted, and processed. His contributions have left an enduring legacy, cementing his place as one of the most influential figures in the history of science and technology. Read more in Wikipedia.
Reference: [1, 2, 3].

References

[1] C. E. Shannon, "A mathematical theory of communication," *The Bell system technical journal*, vol. 27, no. 3, pp. 379–423, 1948.

[2] ——, "A symbolic analysis of relay and switching circuits," *Transactions of the American Institute of Electrical Engineers*, vol. 57, no. 12, pp. 713–723, 1938.

[3] ——, "Communication theory of secrecy systems," *The Bell System Technical Journal*, vol. 28, no. 4, pp. 656–715, 1949.